Review of Math 231

Homework packet for Math 232 Sections 01 and 02

If you want to succeed in Math 232, you first have to make sure that you have a solid foundation from Math 231. This homework packet will help you review key material from Math 231 and make sure that everyone starts Math 232 with the same basic knowledge.

The double Homework Quiz on Wednesday, January 18 will consist of problems very similar to the ones in this packet. Remember that you can write anything you like in your Math 232 Homework Notebook, and that you will be able to use your Math 232 Homework Notebook (but not your old Math 231 Notebook, if you have one) on the double Homework Quiz.

Part 1. Basic Overview

The Chapter Review exercises in the book will help you review basic definitions, theorems, notation, and skills from the material in Math 231. Complete each Chapter Review and check off each part as you go.

Chapter 0. Review, Self-Test, and Capstones (pp.89–91)

    ____ Definitions
    ____ Theorems
    ____ Notation and Algebraic Rules
    ____ Skill Certification: Basic Algebra and Simple Functions
    ____ Capstone Problems

Chapter 1. Review, Self-Test, and Capstones (pp.161–162)
(Don't worry about questions on delta-epsilon proofs from Section 1.3, but do make sure you understand questions about the formal definition of limit from Section 1.2)

    ____ Definitions
    ____ Theorems
    ____ Limit Rules and Indeterminate Forms
    ____ Skill Certification: Basic Limits
    ____ Capstone Problems

Chapter 2. Review, Self-Test, and Capstones (pp.220–222)

    ____ Definitions
    ____ Theorems
    ____ Notation and Differentiation Rules
    ____ Skill Certification: Basic Derivatives
    ____ Capstone Problems

Chapter 3. Review, Self-Test, and Capstones (pp.285–286)
(Don't worry about questions about Rolle's Theorem or the Mean Value Theorem from Section 3.1)

     _____ Definitions
     _____ Theorems
     _____ Geometric Formulas and Theorems
     _____ Skill Certification: Derivatives and Curve Sketching
     _____ Capstone Problems

Chapter 4. Review, Self-Test, and Capstones (pp.337–339)

     _____ Definitions
     _____ Theorems
     _____ Notation and Algebraic Rules
     _____ Skill Certification: Algebra, Limits, and Basic Graphs
     _____ Capstone Problems

Part 2. Typical Problems from Math 231

Calculate each of the following and record your answers in the blanks. Write out your work clearly and in order in your Notebook so that these problems can serve as examples when you need them.

1. Write the domain of $f(x) = \sqrt{x^2 - 9}$ in interval notation.

2. Write the domain of $f(x) = \dfrac{\sqrt{x+2}}{x(x-3)}$ in interval notation.

3. If f is an invertible function with $f(8) = 2$, what is $f^{-1}(2)$?

4. Use the definition of derivative (not the "shortcut rules") to find the derivative of $f(x) = x^3$.

5. Use the definition of derivative (not the "shortcut rules") to find the derivative of $f(x) = \frac{1}{x}$.

6. Express the solution set of the inequality $0 < |x - 3| < 0.5$ in interval notation.

7. Use a number line to determine the intervals on which $f(x) = \dfrac{x - 1}{x(x + 4)}$ is positive, and those on which it is negative.

8. Use a number line to determine the intervals on which $f(x) = x^4 - 6x^2 - 8x + 2$ is increasing, and those on which it is decreasing.

9. Use a number line to determine the intervals on which $f(x) = x^3 - 3x^2 + x + 4$ is concave up, and those on which it is concave down.

10. If $f(x) = \dfrac{x^2 - 4x + 4}{x^2 - 5x + 6}$, define a value for $f(2)$ so that f is continuous at $x = 2$.

11. A stone is thrown straight down from the edge of a roof, 1150 feet above the ground, at a speed of 9 feet per second. Remembering that the acceleration due to gravity is -32 feet per second squared, at what time does the stone hit the ground, and what is its velocity at the moment of impact?

12. If $f'(x) = 2x + 1$ and $f(0) = 5$, then find $f(x)$.

13. Use wolframalpha.com to sketch a graph of the function $y = x^2(x - 3)$, and copy that graph onto paper, and imagine that the graph you just sketched is the graph of the derivative $f'(x)$ of some function $f(x)$. Now use this graph of $f'(x)$ to sketch graphs of the functions $f(x)$ and $f''(x)$.

14. If $f(-2) = 5$ and $f(x)$ is an even function, then what is $f(2)$, and why? What if $f(x)$ is an odd function?

15. Find the global maximum value and the global minimum value for the function $f(x) = 2x^3 + 21x^2 + 72x + 6$ on the interval $[-6, -1]$.

16. A landowner needs to enclose a rectangular space with total area of 775 square feet immediately next to a river. If the river does not require any fencing along that edge, what is the least amount of fencing necessary to enclose the area?

17. Use polynomial long division to write $f(x) = \dfrac{x^5 - 20}{x^2 - 4x + 1}$ as the sum of a polynomial and a proper rational function.

18. Find all roots, holes, and asymptotes (vertical, horizontal, slant, or curve) of the function $f(x) = \dfrac{(x + 7)(x - 4)^2}{(x - 4)(x + 1)}$.

19. Rewrite the function $f(x) = |2x + 9|$ as a piecewise function that does not involve absolute values.

20. If $f(x) = x^{\frac{1}{3}} + 1$, then find $f^{-1}(x)$ and describe its domain and range.

21. If $f(x) = \sqrt{x + 1}$, then find $f^{-1}(x)$ and describe its domain and range.

22. If $f(x)$ and $g(x)$ are inverses of each other, then what can you say about $f(g(x))$?

23. Air is being pumped into a spherical balloon so that its volume increases at a rate of 80 cubic centimeters per second. How fast is the surface area of the balloon increasing when its radius is 7 centimeters?

Part 3. Limits Practice

Calculate each of the following limits algebraically, showing all work clearly. Afterwards, verify your answers graphically with wolframalpha.com.

1. $\lim\limits_{x \to 0} 2x^{\frac{-3}{4}}$

2. $\lim\limits_{x \to 2} \dfrac{4 + 2x}{x + 2}$

3. $\lim\limits_{x \to 1} \dfrac{1}{x^2 - 1}$

4. $\lim\limits_{x \to -\infty} (5 - 2x + 3x^3)$

5. $\lim\limits_{x \to \infty} 4x^{-3}$

6. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x}}{1 - \sqrt{x}}$

7. $\lim\limits_{x \to \infty} \dfrac{(3x + 1)^2(x - 1)}{(1 - x^3)}$

8. $\lim\limits_{x \to 0} (3x^{-1} - 2x^{-2})$

9. $\lim\limits_{x \to 2} \dfrac{x + 1}{(x - 2)^2}$

10. $\lim\limits_{x \to 1} \dfrac{x - 1}{x^2 - 2x + 1}$

11. $\lim\limits_{x \to \infty} (\sqrt{x} - x)$

12. $\lim\limits_{x \to 1} \dfrac{x + x^2 - 2x^3}{x - x^2}$

13. $\lim\limits_{h \to 0} \dfrac{(2 + h)^3 - 2^3}{h}$

14. $\lim\limits_{h \to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h}$

15. $\lim\limits_{x \to \infty} (-2x^3 + x^2 - 10)$

16. $\lim\limits_{x \to \infty} \dfrac{x^{-3}}{x^2 - x^{-1}}$

Part 4. Derivatives Practice

For each function $f(x)$ find $f'(x)$. Check your answers on wolframalpha.com.

1. $f(x) = \sqrt{(3x^4 - 1)^3}$

2. $f(x) = \sqrt{(3x^4 - 1)^3 + x}$

3. $f(x) = \dfrac{\sqrt{1-x}}{x^2 - 4}$

4. $f(x) = \sqrt{x}(5x + 2)^{100}$

5. $f(x) = \sqrt{x(5x + 2)^{100}}$

6. $f(x) = (\sqrt{x}(5x + 2))^{100}$

7. $f(x) = \dfrac{x^5 + x\sqrt{x}}{x^2}$

8. $f(x) = \dfrac{1}{\sqrt{x}} + \dfrac{1}{x^2}$

9. $f(x) = \sqrt{\sqrt{x}}$

10. $f(x) = \dfrac{3}{x^{-\frac{3}{2}}\sqrt{x}}$

11. $f(x) = (3x + 1)^2(2x + 3)^8(5x - 2)^4$

12. $f(x) = \dfrac{(x - 1)(x - 2)}{(x - 3)(x - 4)}$

13. $f(x) = (x^2 - 17x)^{-9} \cdot \dfrac{x^2 + 1}{\sqrt{2x + 1}}$

14. $f(x) = (((x^2 + 1)^2 + 1)^2 + 1)^2$

15. $3x^2 + 4y^2 + xy = 0$ (find $\frac{dy}{dx}$)

16. $\dfrac{y^3 + 1}{x^3 + 1} = y^2$ (find $\frac{dy}{dx}$)

17. $\dfrac{1}{y} - \dfrac{1}{x} = \dfrac{x^3}{y - 1}$ (find $\frac{dy}{dx}$)

18. $A(t) = \pi(r(t))^2$ (find $\frac{dA}{dr}$ and $\frac{dA}{dt}$)

19. $f(x) = \dfrac{1}{x^2 + 1}$
(find $f'(x)$, $f''(x)$, and $f'''(x)$)

20. $f(x) = 10x^8 + 6x^5 - 4x^2 + 17$
(find $f'(x)$ and $f^{[8]}(x)$ and $f^{[9]}(x)$)